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The Number of Fuzzy Subgroups of Rectangle Groups

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Abstract

In this paper, we construct the formula to determine the number of fuzzy subgroups of finite groups, which are their lattices have a pattern. We discuss for the special pattern, that is "rectangle". First, we determine for rectangle $2 \times n$, then we get for 3×4. We also explain the applications of that formula for some groups.We can see that the result of Bashir Humera and Zahid Raza in [1] about the number of fuzzy subgroups of $Z_{p^n \times} Z_q$ is special case of this formula.

Keywords: Fuzzy Subgroups, Lattice, Lattice Method

1 Introduction

One of the most important problem of fuzzy group theory is to classify the fuzzy subgroup of a finite group. Many papers have treated the particular case of finite cyclic group. Laszlo [2] studied about the construction of fuzzy subgroup of group of order one to six. Zhang and Zou [3] have determined the number of fuzzy subgroup of cyclic groups of order p^n where p is a prime number. Murali and Makamba ([4], [5]) considered a similar problem. They have determined the number of fuzzy subgroups of $Z_{p^n}, Z_p \times Z_q, Z_{p^n} \times Z_q$ and abelian groups of order $p^n q^m$ where p and q are different primes. Tarnauceanu and Bentea [6] have determined this number for finite abelian groups. Sulaiman and Abd Ghafur [7] have determined the number of fuzzy subgroups of finite cyclic groups. Sulaiman [8] has constructed fuzzy subgroups of Symmetric group S_4.

The interesting result of Sulaiman [9] is a method to determine the number of fuzzy subgroup of finite groups. This method called "Lattice Method". By using this method Sulaiman [9] has counted the number of fuzzy subgroups of $p-group$ (Theorem 18), the number of fuzzy subgroups of $Z_p \times Z_q$ (Theorem 21) and the number of fuzzy subgroups of $Z_{p^n} \times Z_q$ (Theorem 23).

In this article, we use that method to construct some formulas to determine the number of fuzzy subgroups of finite groups which their lattices have a pattern. That pattern is "rectangle".We have determined a formulas for rectangle "$2 \times n$".

2 Preliminary

We recall some definitions and results that will be used later. Proving of some theorems in this section can be written in [9].

Definition 2.1 Let X be a nonempty set. A fuzzy subset of X is a function from X into [0, 1].

Definition 2.2 (*Rosenfeld, see* [10]) Let G be a group. A fuzzy subset μ of G is called a fuzzy subgroup of G if

(1) $\mu(xy) \geq min\{\mu(x), \mu(y)\}, \forall x, y \in G$

(2) $\mu(x^{-1}) \geq \mu(x), \forall x \in G$.

Theorem 2.3 (*Rosenfeld, see* [10]) Let e denote the identity element of group G. If μ is a fuzzy subgroup of G, then

(1) $\mu(e) \geq \mu(x), \forall x \in G$

(2) $\mu(x^{-1}) = \mu(x), \forall x \in G$.

Theorem 2.4 (*Sulaiman and Abdul Ghafur, see* [11]) A fuzzy subset μ of G is a fuzzy subgroup if and only if there is a chain of subgroups G, $P_1(\mu) \leq P_2(\mu) \leq \ldots \leq P_m(\mu) = G$ such that μ is in the form

$$\mu(x) = \begin{cases} \theta_1, x \in P_1(\mu) \\ \theta_2, x \in P_2(\mu) \\ \vdots \\ \theta_m, x \in P_m(\mu) \end{cases} \quad (1)$$

If there is no confusion, then $P_i(\mu)$ as in (1) can simply be written as P_i. We define *length* (or *order*) of fuzzy subgroup μ in (1) as be m.

Theorem 2.5 *(Sulaiman, see* [9]) Let $G_1 < G_2 < \cdots < G_{k-1} < G_k = G$ be the only maximal chain from G_1 to G on the lattice subgroups of G. Then $n(F_{P_1=G_1}) = \begin{cases} 1, k = 1,2 \\ 2^{k-2}, k > 2 \end{cases}$.

Theorem 2.6 *(Sulaiman, see* [9]) Let $G_1 < G_2 < \cdots < G_{k-1} < G_k = G$ be the only maximal chain from G_1 to G on the lattice subgroups of G. For m, $1 \leq m \leq m-2$ we have $n(F_{P_1=G_m}) = 2.n(F_{P_1=G_{m+1}})$.

Theorem 2.7 *(Sulaiman, see Theorem 11 in* [9]) Let H be a subgroup of G, and let the set of all subgroups of G which contain H (but are not equal to H) be $\{H_1, H_2, \dots, H_k\}$. Then $n(F_{P_1=H}) = \sum_{i=1}^{k} n(F_{P_1=H_i})$.

Corollary 2.8 *(Sulaiman, see Corollary 14 in* [8]) Let G be a group. Then $n(F_G) = 2.n(F_{P_1=\{e\}})$.

3 Main Result

Definition 3.1 Let G be a group and the number of it's subgroups is finite. The diagram of lattice subgroups G is called "rectangle" if satisfies these conditions: The subgroups of G can be labeled by K_j^i where $1 \leq i \leq m$, $1 \leq j \leq s$ for some $m, s \in Z$ with $K_1^1 = G$, $K_s^m = \{e\}$ such that : (i) for fixed i, $1 \leq i \leq m, K_k^i < K_{k-1}^i, \forall k, 2 \leq k \leq s$, (ii) for fixed j, $1 \leq j \leq m$, $K_j^i < K_j^{i-1}, \forall t, 2 \leq t \leq m$. In this case the size of the diagram is $m \times s$. The diagram of "rectangle lattice" is shown in Figure 1.

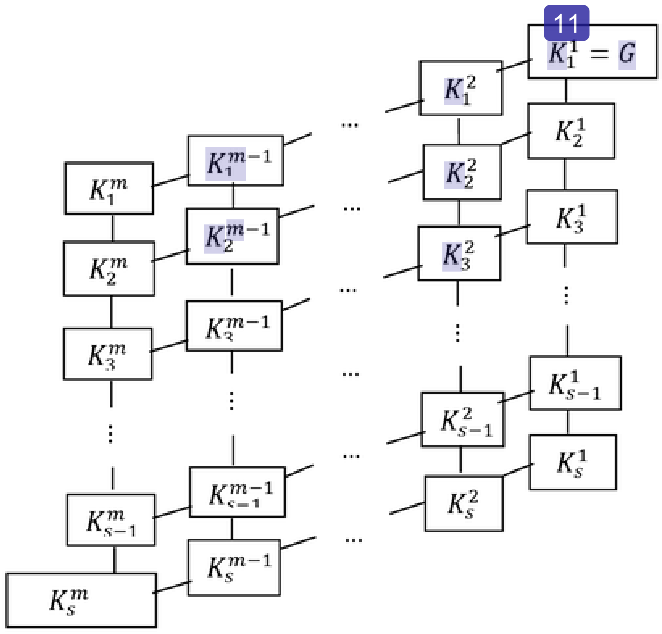

Figure 1: Rectangle lattice

Theorem 3.2 Let G be a group that satisfied Definition 3.1 with $m = 2, s \in$ *N*. We have:

$i)$ $n(F_{P_1=K_s^2=\{e\}}) = 2.n(F_{P_1=K_{s-1}^2}) + n(F_{P_1=K_s^1})$

$ii)$ $n(F_{P_1=K_s^2=\{e\}}) = 2^s + (s-3)2^{s-2} = 2^{s-2}(s+1)$.

Proof. Consider the diagram of G (see Figure 2).

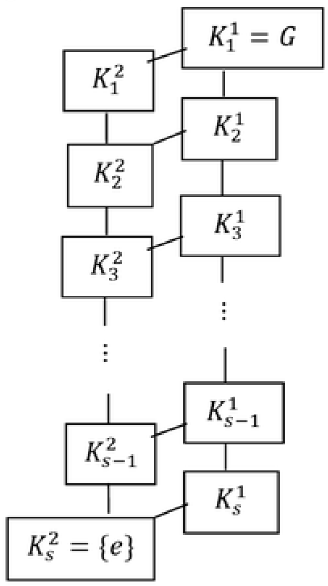

Figure 2:Lattice diagram for $2 \times s$

i) According to the Theorem 2.7, we have

$$n\left(F_{P_1=K_s^2}\right) = \sum_{i=1}^{s-1} n\left(F_{P_1=K_i^2}\right) + \sum_{j=1}^{s} n\left(F_{P_1=K_j^1}\right)$$

$$= \sum_{i=1}^{s-1}\left[n\left(F_{P_1=K_i^1}\right) + n\left(F_{P_1=K_i^2}\right)\right] + n\left(F_{P_1=K_s^1}\right) \tag{2}$$

$$n\left(F_{P_1=K_{s-1}^2}\right) = \sum_{i=1}^{s-2} n\left(F_{P_1=K_i^2}\right) + \sum_{j=1}^{s-1} n\left(F_{P_1=K_j^2}\right)$$

$$= \sum_{i=1}^{s-2}\left[n\left(F_{P_1=K_i^1}\right) + n\left(F_{P_1=K_i^2}\right)\right] + n\left(F_{P_1=K_{s-1}^1}\right) \tag{3}$$

Equality (2) can be write as

$$n\left(F_{P_1=K_s^2}\right) = \sum_{i=1}^{s-1}\left[n\left(F_{P_1=K_i^1}\right) + n\left(F_{P_1=K_i^2}\right)\right] + n\left(F_{P_1=K_s^1}\right)$$

$$= \sum_{i=1}^{s-2}\left[n\left(F_{P_1=K_i^1}\right) + n\left(F_{P_1=K_i^2}\right)\right] + n\left(F_{P_1=K_{s-1}^1}\right) + n\left(F_{P_1=K_{s-1}^2}\right) + n\left(F_{P_1=K_s^1}\right) \tag{4}$$

Substitute (3) to (4) to get

$$n\left(F_{P_1=K_s^2}\right) = n\left(F_{P_1=K_{s-1}^2}\right) + n\left(F_{P_1=K_{s-1}^2}\right) + n\left(F_{P_1=K_s^2}\right)$$

$$= 2.\, n\left(F_{P_1=K_{s-1}^2}\right) + n\left(F_{P_1=K_s^2}\right)$$

ii) We proof by induction on s. Using Theorem 2.6 and Theorem 2.7 we have $n\left(F_{P_1=K_s^2}\right) = 1$. Therefore, the statement is true for $s = 1$. Assume that the statement is true for $s = t$. It means $n\left(F_{P_1=K_s^2=\{e\}}\right) = 2^t(t-3)2^{s-2}$. For $s = t+1$, the diagram of lattice subgroup of G is shown in Figure 3.

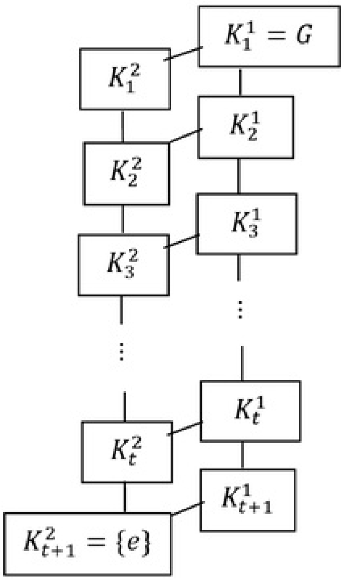

Figure 3: Lattice for $s = t + 1$

Using Theorem 3.2 i) and equality (4) we have

$$n\left(F_{P_1=K_{t+1}^2=\{e\}}\right) = 2.n\left(F_{P_1=K_t^2}\right) + n\left(F_{P_1=K_{t+1}^1}\right)$$

$$= 2[2^t + (t-3)2^{t-2}] + n\left(F_{P_1=K_{t+1}^1}\right).$$

By using Theorem 2.4 we have

$$n\left(F_{P_1=K_{t+1}^2=\{e\}}\right) = 2^{t+1} + (t-3)2^{t-1} + 2^{t-1} = 2^{t+1} + (t-1)2^{t-1}.$$

Analogue with this proving, we have the next theorem.

Theorem 3.3 Let G be a group that satisfied Definition 3.1 with $m \in N, s = 2$. We have:

$i)\ n\left(F_{P_1=K_2^m}\right) = 2.n\left(F_{P_1=K_2^{m-1}}\right) + n\left(F_{P_1=K_1^m}\right)$

$ii)\ n\left(F_{P_1=K_2^m}\right) = 2^m + (m-3)2^{m-2} = 2^{m-2}(m+1)$

Theorem 3.4 Consider group $G = Z_{p^n} \times Z_q$. The diagram lattice subgroup of G is shown as Figure 4. The diagram is satisfy Definition 3.1 with $m = n+1$ and $s = 2$. By applying Theorem 3.2 and Corollary 2.8 we have the number of fuzzy subgroups of $G = Z_{p^n} \times Z_q$ is $2^n(n+2)$.

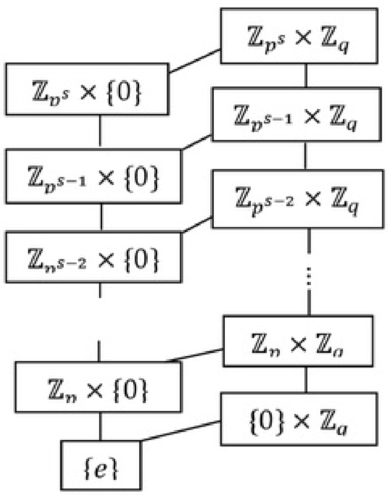

Figure: Lattice Subgroup of $Z_{p^n} \times Z_q$

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